Full-coupled channel approach to $S = -2$ s-shell hypernuclear systems

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We describe full-coupled-channel *ab initio* calculations among the octet baryons for $S = -2$ s-shell hypernuclei, ${}_{\Lambda\Lambda}^{4}$H, ${}_{\Lambda\Lambda}^{5}$H and ${}_{\Lambda\Lambda}^{6}$He. The wave function includes $\Lambda\Lambda$, $\Lambda\Sigma$, $N\Xi$ and $\Sigma\Sigma$ channels. Minnesota NN, D2′ YN and Nijmegen model D simulated YY interactions are used. This is the first attempt to explore the few-body problem of the full-coupled channel scheme for $A = 4-6$, $S = -2$ multistrangeness hypernuclear systems. Bound state solutions of the $\Lambda\Lambda$ hypernuclei, ${}_{\Lambda\Lambda}^{4}$H, ${}_{\Lambda\Lambda}^{5}$H and ${}_{\Lambda\Lambda}^{6}$He, are obtained.

1. INTRODUCTION — Why full-coupled channel? —

Both experimental and theoretical searches for ${}_{\Lambda\Lambda}^{4}$H are of utmost interest in the field of hypernuclei[1,2,3,4,5,6]. On the theoretical side, one of the conclusions in our recent publication[4] is that, "${}^3S_1$ ΛN *interaction has to be determined very carefully, since* $B_{\Lambda\Lambda}$ *is sensitive to the* ${}^3S_1$ *channel of the* ΛN *interaction.* " Our standpoint can be explained by examining the relative importance of ${}^3S_1$ and ${}^1S_0$ ΛN and of ${}^1S_0$ $\Lambda\Lambda$ interactions in a simple *core nucleus* + 2Λ model:

$$\left\langle \sum_{i=1}^{N}\sum_{j=1}^{Y} v_{N_i\Lambda_j} \right\rangle = \mathcal{N}_t\ \bar{v}_{t,N\Lambda} + \mathcal{N}_s\ \bar{v}_{s,N\Lambda}, \qquad \left\langle \sum_{i<j}^{Y} v_{\Lambda_i\Lambda_j} \right\rangle = n_s\ \bar{v}_{s,\Lambda\Lambda}, \tag{1}$$

where N (Y) is the number of nucleons (hyperons) and thus $A = N + Y$. $\mathcal{N}_t$, $\mathcal{N}_s$ and n_s are the number of ${}^3S_1$ ΛN, ${}^1S_0$ ΛN and ${}^1S_0$ $\Lambda\Lambda$ pairs, respectively. Each $\bar{v}$ is the average of the potential matrix element in the appropriate channel. Table 1 lists the number of pairs, $\mathcal{N}_t$, $\mathcal{N}_s$ and n_s (right side). The table also lists the numbers for $S = -1$ hypernuclei (left side). In the $S = -2$ systems, the number n_s is always 1, and the ratio between $\mathcal{N}_t$ and $\mathcal{N}_s$ is always $\mathcal{N}_t : \mathcal{N}_s = 3 : 1$. This means that the $B_{\Lambda\Lambda}$ binding energy strongly depends on the ${}^3S_1$ ΛN interaction than the ${}^1S_0$ ΛN or the ${}^1S_0$ $\Lambda\Lambda$ interaction. Particularly, in searching for ${}_{\Lambda\Lambda}^{4}$H, a check of the ΛN potential concerning the observed binding energy of only the subsystem, ${}_{\Lambda}^{3}$H, is insufficient, since the $B_\Lambda({}_{\Lambda}^{3}\mathrm{H})$ strongly depends on the ${}^1S_0$ ΛN interaction than the ${}^3S_1$ ΛN interaction. The algebraic structure of the ΛN pairs for $S = -2$ systems is very similar to the structure of ${}_{\Lambda}^{5}$He in $S = -1$ systems. This implies

Table 1
Numbers of $^3S_1$ and $^1S_0$ ΛN and $^1S_0$ $\Lambda\Lambda$ pairs for $S=-1$ (left side) or $S=-2$ (right side) hypernucleus in a "*core nucleus* + Λ" or a "*core nucleus* + 2Λ" model.

$S=-1$	$\mathcal{N}_t$	$\mathcal{N}_s$
$^3_\Lambda$H	$\frac{1}{2}$	$\frac{3}{2}$
$^4_\Lambda$H, $^4_\Lambda$He	$\frac{3}{2}$	$\frac{3}{2}$
$^4_\Lambda$H*, $^4_\Lambda$He*	$\frac{5}{2}$	$\frac{1}{2}$
$^5_\Lambda$He	3	1

$S=-2$	$\mathcal{N}_t$	$\mathcal{N}_s$	n_s
$_{\Lambda\Lambda}^{\ 4}$H	3	1	1
$_{\Lambda\Lambda}^{\ 5}$H, $_{\Lambda\Lambda}^{\ 5}$He	$\frac{9}{2}$	$\frac{3}{2}$	1
$_{\Lambda\Lambda}^{\ 6}$He	6	2	1

that the ΛN interaction utilized in the study of $S=-2$ hypernuclei should reproduce the $B_\Lambda(^5_\Lambda\mathrm{He})$ as well as the B_Λ's of $A=3,4$ $S=-1$ hypernuclei. However, a single channel ΛN potential, which reproduces the B_Λ values of $A=3,4$ $S=-1$ hypernuclei as well as the Λp total cross section, cannot reproduce the experimental $B_\Lambda(^5_\Lambda\mathrm{He})$ value. This is known as a $^5_\Lambda$He anomaly, which was the long standing problem since the publication in 1972 by Dalitz *et al.*[7]. According to the recent study by Akaishi *et al.*[8], Σ degrees of freedom have to be explicitly taken into account so as to resolve the $^5_\Lambda$He anomaly.

Considering the fact that the $\Lambda\Lambda$ system couples to $N\Xi$ and $\Sigma\Sigma$ states, as well as the ΛN couples to the ΣN, a theoretical search for $_{\Lambda\Lambda}^{\ 4}$H should be made in a full-coupled channel formulation with a set of interactions among the octet baryons. Thus the purpose of this study is to describe a systematic study for s-shell $\Lambda\Lambda$ hypernuclei in a framework of fully coupled channel formulation.

2. INTERACTIONS AND METHOD

The wave function of a system with strangeness $S=-2$, comprising A octet baryons, has four isospin-basis components. For example, $_{\Lambda\Lambda}^{\ 6}$He has four components as $ppnn\Lambda\Lambda$, $NNNNN\Xi$, $NNNN\Lambda\Sigma$ and $NNNN\Sigma\Sigma$. We abbreviate these components as $\Lambda\Lambda$, $N\Xi$, $\Lambda\Sigma$ and $\Sigma\Sigma$, referring the last two baryons. The hamiltonian of the system is hence given by 4×4 components as

$$H=\begin{pmatrix} H_{\Lambda\Lambda} & V_{N\Xi-\Lambda\Lambda} & V_{\Lambda\Sigma-\Lambda\Lambda} & V_{\Sigma\Sigma-\Lambda\Lambda} \\ V_{\Lambda\Lambda-N\Xi} & H_{N\Xi} & V_{\Lambda\Sigma-N\Xi} & V_{\Sigma\Sigma-N\Xi} \\ V_{\Lambda\Lambda-\Lambda\Sigma} & V_{N\Xi-\Lambda\Sigma} & H_{\Lambda\Sigma} & V_{\Sigma\Sigma-\Lambda\Sigma} \\ V_{\Lambda\Lambda-\Sigma\Sigma} & V_{N\Xi-\Sigma\Sigma} & V_{\Lambda\Sigma-\Sigma\Sigma} & H_{\Sigma\Sigma} \end{pmatrix}, \tag{2}$$

where $H_{B_1B_2}$ operates on the B_1B_2 component, and $V_{B_1B_2-B_1'B_2'}$ is the sum of all possible two-body transition potential connecting B_1B_2 and $B_1'B_2'$ components.

In the present calculation, we use Minnesota potential for the NN interaction, D2$'$ for the YN and Nijmegen model D simulated (ND(S)) for the YY interaction. The Minnesota potential reproduces reasonably well both the binding energies and sizes of few-nucleon systems, such as ^{2}H, ^{3}H, ^{3}He and ^{4}He[9]. The D2$'$ potential is a modified potential from the original D2 potential[8]. The strength of the long-range part (V_b in Table I of Ref. [8]) of the D2$'$ potential in the ΛN-ΛN $^3S_1$ channel is reduced by multiplying a factor 0.954, in order to reproduce the experimental $B_\Lambda(^5_\Lambda\mathrm{He})$ value. We take the hard core radius as

Table 2
Parameters of the ND(S) YY potential in the even states, given in units of MeV.

		^1E		^3E	
		v_S	v_L	v_S	v_L
$\Lambda\Lambda{-}\Lambda\Lambda$	$I=0$	1464.48	-95.00		
$\Lambda\Lambda{-}N\Xi$	$I=0$	200.86	15.57		
$N\Xi{-}N\Xi$	$I=0$	1205.68	-89.19	968.12	-70.28
	$I=1$	1783.17	-66.54	862.75	-72.23
$\Lambda\Lambda{-}\Sigma\Sigma$	$I=0$	648.96	-42.81		
$N\Xi{-}\Lambda\Sigma$	$I=1$	103.84	62.98	-311.72	-42.42
$N\Xi{-}\Sigma\Sigma$	$I=0$	-301.11	87.11		
	$I=1$			-14.18	-8.26
$\Lambda\Sigma{-}\Lambda\Sigma$	$I=1$	1245.45	-90.67	909.99	-85.40
$\Lambda\Sigma{-}\Sigma\Sigma$	$I=1$			-129.02	-41.88
$\Sigma\Sigma{-}\Sigma\Sigma$	$I=0$	350.73	-15.32		
	$I=1$			738.86	-74.71
	$I=2$	693.54	-114.26		

$r_c = 0.56(0.45)$ fm for the ND YY interaction in the $^1S_0(^3S_1)$ channel, which is the same as the hard core radius of the YN sector. The ND(S) potential is given by

$$v(r) = v_S \exp\{-(r/\beta_S)^2\} + v_L \exp\{-(r/\beta_L)^2\}, \qquad (3)$$

with $\beta_S = 0.5$ fm and $\beta_L = 1.2$ fm. The strength parameters are shown in Table 2.

The calculations were made by using stochastic variational method[9]. The reader is referred to Ref. [10] for the details of the method.

3. RESULTS AND DISCUSSIONS

Table 3 lists the B_Λ and $B_{\Lambda\Lambda}$ values for $S=-1$ and -2 hypernuclei. The D2′ YN potential well reproduces all the B_Λ values for $A=3-5$, $S=-1$ hypernuclei. The D2′ includes explicit $\Lambda N - \Sigma N$ coupling, which is significantly important to resolve the ${}^5_\Lambda$He anomaly.

Using the ND(S) YY potential, we have obtained the bound state solutions of ${}_{\Lambda\Lambda}^{\ 4}$H,

Table 3
Λ and $\Lambda\Lambda$ separation energies, given in units of MeV, of $A=3-6$, $S=-1$ and -2 s-shell hypernuclei.

	$B_\Lambda({}^3_\Lambda\mathrm{H})$	$B_\Lambda({}^4_\Lambda\mathrm{H})$	$B_\Lambda({}^4_\Lambda\mathrm{H}^*)$	$B_\Lambda({}^5_\Lambda\mathrm{He})$	$B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{\ 4}\mathrm{H})$	$B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{\ 5}\mathrm{H})$	$B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{\ 6}\mathrm{He})$
Calc	0.056	2.23	0.91	3.18	0.107	4.04	7.93
Exp	0.13(5)	2.04(4)	1.00(4)	3.12(2)			7.3(3)

Table 4
Probabilities, given in percentage, of $N\Xi$, $\Lambda\Sigma$ and $\Sigma\Sigma$ components for $\Lambda\Lambda$ hypernuclei, ${}^{A}_{\Lambda\Lambda}$Z. The probabilities of the Σ component for the subsystems, ${}^{A-1}_{\Lambda}$Z, are also given in parentheses.

	${}^{4}_{\Lambda\Lambda}$H (${}^{3}_{\Lambda}$H)	${}^{5}_{\Lambda\Lambda}$H (${}^{4}_{\Lambda}$H, ${}^{4}_{\Lambda}$H*)	${}^{6}_{\Lambda\Lambda}$He (${}^{5}_{\Lambda}$He)
$P_{N\Xi}$	0.12	4.34	0.27
$P_{\Lambda\Sigma}$ (P_{Σ})	0.35 (0.16)	2.52 (2.17, 0.36)	1.18 (0.55)
$P_{\Sigma\Sigma}$	0.01	0.05	0.04

${}^{5}_{\Lambda\Lambda}$H and ${}^{6}_{\Lambda\Lambda}$He. The obtained $B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})$ is slightly larger than the experimental value; $\Delta B^{(\mathrm{calc})}_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) = 1.58$ MeV, while $\Delta B^{(\mathrm{exp})}_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) = 1.01 \pm 0.20^{+0.18}_{-0.11}$ MeV[11]. The calculated $B_{\Lambda\Lambda}({}^{4}_{\Lambda\Lambda}\mathrm{H}) = 0.107$ MeV is very close to but lower than the obtained ${}^{3}_{\Lambda}\mathrm{H} + \Lambda$ threshold, $B_{\Lambda}({}^{3}_{\Lambda}\mathrm{H}) = 0.056$ MeV.

Table 4 lists the probabilities of Σ for $S = -1$ and of $N\Xi$, $\Lambda\Sigma$ and $\Sigma\Sigma$ components for $S = -2$ hypernuclei. In the present calculation, the $\Lambda\Lambda$ component is the main part of the wave function of the $S = -2$ hypernuclei. No unrealistic bound states were found, since the hard core model hardly incorporates unrealistic strong attractive force in the short range region, in contrast to the soft core model such as NSC97e or NSC97f[12]. This is one of the reasons why we used the hard core model for the YY interaction for the first attempt to the full-coupled channel calculation. $P_{\Sigma\Sigma}$'s are very small for both systems, due to large mass difference between $\Lambda\Lambda$ and $\Sigma\Sigma$ channels ($m_{\Sigma\Sigma} - m_{\Lambda\Lambda} \cong 155$ MeV).

The $P_{N\Xi}({}^{5}_{\Lambda\Lambda}\mathrm{H})$ is a surprisingly large amount, in spite of the fact that the strength of the $\Lambda\Lambda - N\Xi$ coupling of ND is not strong. This implies that the coupling between $t + \Lambda + \Lambda$ channel and $\alpha + \Xi^-$ channel (α formation effect[13]) is significant.

We should note that the present results depend on the choice of the YY potential. If we use other YY potential model, the quantitative results must change. More comprehensive study using various kinds of YY interaction models will be reported in the near future.

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